Written Round: February 25, 2023 at Regional Testing Sites

Ciphering Round: April 15, 2023 at University of North Alabama

ALGEBRA II EXAMINATION

Construction of this test directed
by
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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E. A calculator is NOT permitted.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a \perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage0ri7 17e(the)-422a2ne the4(remainpherate)]TJ 0

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

Market Research Analyst	Cryptanalyst	Mathematician
Air Traffic Controller	Professor	Meteorologist
Climate Analyst	Pollster	Medical Doctor
Estimator	Population Ecologist	Lawyer
Research Scientist	Operations Research	Actuary
Computer Programmer	Data Mining	Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

U.S. Government Agencies such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.

Government labs and research offices such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.

Engineering research organizations such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.

Computer information and software firms such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.

Electronics and computer manufacturers such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.

Aerospace and transportation equipment manufacturers such as Boeing, Ford, General Motors, and Lockheed Martin.

Transportation service providers such as FedEx Corporation and United Parcel Service (UPS).

Financial service and investment manangement firms such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)

Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. Find the sum of the x-components of the x-intercepts of the line given by $2x + 3y = 5$ and the line perpendicular to it through the point $(1, 1)$.

   (A) $\dfrac{7}{6}$ (B) $\boxed{\dfrac{17}{6}}$ (C) $\dfrac{23}{6}$ (D) $\dfrac{25}{6}$ (E) None of these

2. If (a, b) is a solution to the system below, find the smallest possible value of $a + b$.

$$\frac{1}{x^2} + \frac{3}{y^3} = 1$$

$$\frac{2}{x^2} - \frac{1}{y^3} = 7$$

   (A) $\boxed{\dfrac{1}{2}}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{3}{2}$ (D) $\dfrac{5}{4}$ (E) None of these

3. The first and fifth terms of an arithmetic sequence are -3 and 17, respectively. Find the 50th term.

   (A) 193   (B) $\boxed{242}$ (C) 247   (D) 977   (E) None of these

4. Simplify $\sqrt[r]{\dfrac{1}{3}} + 5\sqrt{12} - 2\sqrt{108}$.

   (A) $\dfrac{11\sqrt{3}}{3}$ (B) $\boxed{\dfrac{5\sqrt{3}}{3}}$ (C) $\dfrac{13\sqrt{3}}{3}$ (D) $\dfrac{25\sqrt{3}}{3}$ (E) None of these

5. A jar contains $6.65 made up of nickels (value of five cents), dimes (value of ten cents), and quarters (value of twenty five cents). There are the same number of nickels as dimes. If there are 70 total coins, how many more dimes are there than quarters?

   (A) $\boxed{23}$ (B) 25   (C) 29   (D) 31   (E) None of these

6. Which of the following pairs (a, x) is a solution to the equation

$$\frac{4a + x}{a + x} - 1 = \frac{12a}{3a + x}?$$

   (A) $(\sqrt{5}, 3\sqrt{5})$ (B) $(3\sqrt{5}, \sqrt{5})$ (C) $(-\sqrt{5}, 3\sqrt{5})$ (D) $\boxed{(-3\sqrt{5}, \sqrt{5})}$ (E) None of these

7. Find the positive real solution to the equation $2x^2 - 3x^{6 \div 5} - 9x^{2 \div 5} = 0$.

   (A) $\boxed{3\sqrt[4]{3}}$ (B) $\dfrac{3\sqrt[4]{3}}{4}$ (C) $\sqrt[5]{81}$ (D) $\dfrac{\sqrt[5]{162}}{2}$ (E) None of these

8. Find the sum of the reciprocals of the two solutions to the equation $(2x - 3)(x - 4) = 33$.

   (A) $\dfrac{55}{666}$ (B) $\dfrac{39}{74}$ (C) $\dfrac{2}{5}$ (D) $\boxed{\dfrac{11}{21}}$ (E) None of these

9. If a is the largest real solution to the equation $(x^2 - x + 2)^2 + 2(x - 5)(x + 4) = 36$, find $a^3 - 4a + 2$.

   (A) 2   (B) $\boxed{17}$

10. The reciprocals of two numbers sum to 1

30. Use the fact that $\log_{10} 2 \approx 0{:}301$ to find the approximate value of $\log_{10} 50$.

(**A**) 0:7525   (**B**) 1.301   (**C**) 1.505   (**D**) $\boxed{1.699}$ (**E**) None of these

31. Let $f(x) = 2x^3 - 6x^2 + 6x - 18$ and $g(x) = x^2 + 4$. When $x = 80$, what is the positive remainder when $f(x)$ is divided by $g(x)$?

(**A**) 18   (**B**) 154   (**C**) $\boxed{6250}$ (**D**) 6404   (**E**) None of these

32. Find the value of $\left| \dfrac{(5 + i)(2 - 2i)}{3 - 2i} \right|$ where i is the imaginary unit.

(**A**) $\boxed{2}$ (**B**) -2 (**C**) $2 + 2i$ (**D**) $2 - 2i$ (**E**) None of these

33. If $f(x)$ is an even function,

39. Let $p(x$

49. If $x^2 + 2x + 5$ is a factor of $x^4 + px^2 + q$, then what is $p + q$?

(A) 10   (B) 19   (C) 26   (D) 31   (E) None of these

50. Trent is preparing a salad. He can make his salad with just lettuce, or he can make it with lettuce and any of the following toppings: cucumbers, tomatoes, onions, carrots, mushrooms, croutons, and/or cheese. How many variations of the salad are possible?

(A) 49   (B) 64   (C) 128   (D) 256   (E) None of these